# Technical Comments

## Comment on "Temperature Distribution in a Porous Surface"

A. F. Mills* and R. B. Landis†
*University of California, Los Angeles, Calif.*

Recently Libby[1] presented an analysis of the temperature distribution in a porous surface associated with suction of a high-energy boundary-layer flow. Libby concludes that with adequate radiation cooling from the surface, suction for boundary-layer control purposes may provide a source of coolant air. Our purpose is to show that Libby's conclusion is incorrect and is indeed not substantiated by his analysis.

Libby has solved for the fluid and solid temperature distributions across a porous wall subject to the boundary conditions that at $x = 0$ (the exterior surface) the fluid temperature $t_0$ is specified and is equal to the solid temperature $T(0)$, and at $x = L$ the coolant temperature is specified to be $t_c$ where $t_c < t_0$. The results presented in Figs. 2 and 3 (of Ref. 1) represent a valid solution to the problem posed, but do not substantiate assertions made by Libby, namely 1) if the radiative flux from the surface $q_r$ exceeds the convective flux into the surface $q_c$, the ingested fluid is cooled in its passage through the porous wall, and 2) conduction within the solid at the interior surface, compared to the difference $(-q_c + q_r)$, can be neglected.

In order to show that these statements are incorrect we reproduce Eq. (3) of Ref. 1, which is simply the exterior surface energy balance assuming a perfectly opaque surface

$$\lambda_s T'(0^+) = q_r - q_c \tag{1}$$

and a corrected form of Eq. (4), which is the integrated energy conservation equation across the porous wall,

$$\rho v c_p (t_0 - t_c) = -q_c + q_r - \lambda_s T'(L) \tag{2}$$

It is immediately apparent from Eq. (1) that if $q_r > q_c$, then $T'(0^+)$ is positive. Since $T'(0^+)$ is negative in Figs. 2 and 3, the solutions presented by Libby do not correspond to $q_r > q_c$ but rather to $q_r < q_c$. Conduction in the solid at $x = L$ can be related to conduction at $x = 0$ by combining Eqs. (1) and (2) to yield

$$-\lambda_s T'(L) = -\lambda_s T'(0^+) + \rho v c_p (t_0 - t_c) \tag{3}$$

Thus for the situation considered by Libby, the conduction in the solid at $x = L$ is larger than the conduction at $x = 0$, a result which is verified by Figs. 2 and 3 of Ref. 1. Recall from Eq. (1), $\lambda_s T'(0^+) = q_r - q_c$, and hence not only can $\lambda_s T'(L)$ not be neglected compared with $(-q_c + q_r)$, but the absolute magnitude of $\lambda_s T'(L)$ is greater than that of $(-q_c + q_r)$. It follows that the ingested fluid is in fact cooled by removal of heat from the backface of the wall, and in the process an added cooling load of $(q_c - q_r)$ is incurred. It would require less refrigeration if the interior surface were insulated and the ingested fluid cooled at a location removed from the wall.

The problem of suction applied to a high-energy flow for boundary-layer control purposes has a solution of engineering significance if the backface is taken to be insulated. The constants of integration in Eq. (6) of Ref. 1 are then evaluated subject to the boundary conditions

$$\xi = 0, \hat{T} = \hat{t} = 1; \quad \xi = 1, \hat{T}' = 0$$

resulting in the solution

$$\hat{T} = \hat{t} = 1 \text{ for } 0 \leq \xi \leq 1$$

That is, the temperature distributions are uniform throughout the wall, and further, from Eq. (1), $q_c = q_r$ as would be expected.

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* Assistant Professor of Engineering. Associate AIAA.
† Graduate Student. Student Member AIAA.

### Reference

[1] Libby, P. A., "Temperature Distribution in a Porous Surface Involving Either Suction or Injection," *AIAA Journal*, Vol. 7, No. 6, June 1969, pp. 1206–1208.

## Reply by Author to A. F. Mills and R. B. Landis

Paul A. Libby*
*University of California at San Diego, La Jolla, Calif.*

The criticism of Mills and Landis is certainly well founded; we agree that the conclusion in Ref. 1 is not supported by the analysis there because of our disregard of a condition of zero net heat flux on the inner wall, a physically significant condition for our purposes. Fortunately, for other recent work which we have based on the use of suction in high-energy flows and for our confidence in simple physical concepts such as energy conservation it is the analysis that is in error, i.e., our major conclusion regarding suction as a source of coolant is correct and is supported by analysis if carried out properly.

The simple concept we refer to is as follows: If steady-state energy conservation is applied to the gas from $x = 0^-$ to $x \to \infty$ (refer to Ref. 1 for notation) and if there is no radiative flux at $x \to \infty$, i.e., if we take the emissivity of the inner surface to be zero, then conservation of energy applied to the gas implies that

$$\rho v c_p (t_0 - t_c) = -q_c + q_r \tag{1}$$

One solution of this equation is that insisted on by Mills and Landis, i.e., $t_0 = t_c$, $q_c = q_r$, but consider an experiment where $\rho v c_p$, $t_0$, $q_c$, $q_r$ are all specified; are we to believe that Eq. (1) does not apply and does not determine $t_c$?

To support our belief in Eq. (1), we correct our previous analysis by imposing a condition of zero heat flux at the inner surface $x = L$. This necessitates consideration of the conductivity of the gas $\lambda_g$ since we add another boundary condition to those imposed previously. If we restrict our attention to the case of suction $\rho v > 0$,† it is readily clear that for $x > L$, $t \equiv t_c$ and we retain our inner boundary condition $t(L) = t_c$ even though we include conductivity of the gas.

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* Professor, Department of the Aerospace and Mechanical Engineering Sciences. Fellow AIAA.

† Our results for injection should also be corrected but since Mills and Landis commented only on the case of suction we put this matter aside for the time being.

The additional condition of zero heat flux at $x = L$ is thus

$$\lambda_s T'(L^-) + \lambda_g t'(L^-) = 0 \tag{2}$$

provided as above we neglect radiation from the inner surface. If $\lambda_g$ is set equal to zero, then Eq. (2) leads to the Mills-Landis result, $T'(L^-) = 0, t \equiv T \equiv t_0 = t_c$; consideration of $\lambda_g \neq 0$ relieves us of that constraint.

Before proceeding with our analysis we wish to follow two suggestions of D. R. Kassoy regarding this reply. He suggested that we be explicit concerning 1) the role of Eq. (1) in our strategy and 2) the conductive heat flux $q_c$. We consider it convenient in carrying out the analysis to assume that in Eq. (1), $\rho v c_p$, $t_0$, and $t_c$ are specified and thus that a priori only the difference, $q_r - q_c$, is specified. With respect to $q_c$ we note that $q_c = -\lambda_g t'(0^-)$. If we were considering a strictly one-dimensional problem, corresponding to a doubly infinite domain, $x \to \pm\infty$, then the correct boundary conditions would be altered, $q_c$ would not appear explicitly, but $\lambda_g t'(0^-)$ would. However, we are treating as one-dimensional only the semi-infinite region $x \geq 0$; we do so since we consider this a model for the case of practical interest involving a multidimensional, high-speed boundary layer flowing on the external surface $x = 0^-$. In this case we are justified in assuming that $q_c$ is given and that $T(0) = t(0) = t_0$ for both suction and injection.

With Eq. (2) considered we outline our corrected analysis: Eq. (3) of Ref. 1 becomes

$$\rho v c_p t' - \lambda_g t'' = h(T - t) = \lambda_s T'' \tag{3}$$

subject to the conditions of Eq. (4) of Ref. 1 and to Eq. (2) above. An integration of the extremes of Eq. (3), and imposition of the conditions at $x = L[t = t_c, \lambda_s T'(L^-) + \lambda_g t'(L^-) = 0]$ leads to the equation

$$\rho v c_p (t - t_c) = \lambda_g t'(x) + \lambda_s T'(x) \tag{4}$$

One consequence of Eq. (4) is that at $x = 0$, where $t = t_0$, and where the right hand side equals $q_r - q_c$ we recover Eq. (1).

The analysis continues as follows: Differentiation of the first two parts of Eq. (3), elimination of $T'$ by Eq. (4), and nondimensionalization as in Ref. 1 with the added parameter $\epsilon \equiv (\lambda_g/\lambda_s)$ results in a single equation for the distribution of the gas temperature, namely

$$\alpha\epsilon \hat{t}'''(\xi) - \hat{t}'' - \alpha\beta(1 + \epsilon)\hat{t}' + \beta\hat{t} = \beta\hat{t}_c \tag{5}$$

which is to be solved subject to the conditions

$$\hat{t}(0) = 1, \hat{t}(1) = \hat{t}_c, \hat{t}'(0) = \alpha\epsilon\hat{t}''(0)$$

The latter derives from Eq. (3) and from the requirement that $\hat{T}(0) = \hat{t}(0) = 1$. After Eq. (5) is solved, the temperature distribution within the solid is obtained by algebra from Eq. (3) as

$$\hat{T} = \hat{t} + (\alpha\beta)^{-1}\hat{t}' - (\epsilon/\beta)\hat{t}'' \tag{6}$$

The two conditions on $\hat{T}(\xi)$ $\hat{T}(0) = 1, \hat{T}'(1) = -\epsilon\hat{t}'(1)$ have been incorporated in the solution for $\hat{t}(\xi)$.

For the practically interesting case of $\alpha\epsilon \ll 1$ all the roots of the characteristic equation are real; in this case the solution of Eqs. (5) and (6) may be written as

$$(\hat{t} - \hat{t}_c)/(1 - \hat{t}_c) = A_1 e^{\lambda_1\xi} + A_2 e^{\lambda_2\xi} + A_3 e^{\lambda_3}$$

$$(\hat{T} - \hat{t}_c)/(1 - \hat{t}_c) = A_1 e^{\lambda_1\xi}[1 + (\alpha\beta)^{-1}\lambda_1 - (\epsilon/\beta)\lambda_1^2] + A_2 e^{\lambda_2\xi}[1 + (\alpha\beta)^{-1}\lambda_2 - (\epsilon/\beta)\lambda_2^2] + A_3 e^{\lambda_3\xi}[1 + (\alpha\beta)^{-1}\lambda_3 - (\epsilon/\beta)\lambda_3^2]$$

where the $\lambda_i$'s are the roots of the characteristic equation

$$\alpha\epsilon\lambda^3 - \lambda^2 - \alpha\beta(1 + \epsilon)\lambda + \beta = 0$$

and where the $A_i$'s are the arbitrary constants determined by

$$\begin{vmatrix} 1 & 1 & 1 \\ e^{\lambda_1} & e^{\lambda_2} & e^{\lambda_3} \\ \lambda_1(1 - \epsilon\alpha\lambda_1) & \lambda_2(1 - \epsilon\alpha\lambda_2) & \lambda_3(1 - \epsilon\alpha\lambda_3) \end{vmatrix} \begin{vmatrix} A_1 \\ A_2 \\ A_3 \end{vmatrix} = \begin{vmatrix} 1 \\ 0 \\ 0 \end{vmatrix}$$

Perhaps sufficient for our response to the criticism of Mills and Landis and more interesting than results from numerical analysis of this exact solution is the approximate solution which results from letting $\epsilon \to 0$. Some rough calculations based on the exact solution suggest the existence of an inner layer near $\xi = 1$ with an outer solution $\hat{t} \equiv \hat{T} \equiv 1$. Thus we introduce a new inner variable $\tilde{\xi} = (1 - \xi)\epsilon^{-1} > 0$, and find from Eq. (5) that the first order inner solution for $\hat{t}$ is

$$(1 - \hat{t})/(1 - \hat{t}_c) = e^{-\tilde{\xi}/\alpha} = e^{-(1-\xi)/\epsilon\alpha}$$

The first-order inner solution for $\hat{T}$ is found to be $\hat{T} \equiv 1$.

In conclusion we express our appreciation to Mills and Landis for exposing the error in our previous analysis and for forcing us to carry out the correct calculation.

**Reference**

[1] Libby, P. A., "Temperature Distributions in Porous Surfaces Involving Either Suction or Injection," *AIAA Journal*, Vol. 7, No. 6, June 1969, pp. 1206–1207.

# Comments on "Onset of Surface Combustion in Still Atmosphere"

Cye H. Waldman*
*Office National d'Etudes et de Recherches Aérospatiales, Châtillon, France*

IN Ref. 1 Liu has considered the problem of combustion initiation under surface reaction conditions in connection with fire prevention in space capsule interiors. The purpose of this Comment is twofold. First, to demonstrate several errors in Liu's analysis, then to show that related studies, of solid-propellant ignition, have already solved the problem posed in Ref. 1, and more thoroughly.

The governing equations describing the transient behavior of an oxidant containing gas adjacent to a combustible solid surface are the continuity equation, the momentum equation, the species conservation equations, and the heat conduction (or energy) equations for both the solid and gas phases. The momentum equation can generally be neglected in the absence of forced convection† owing to a constant gas phase pressure while the continuity equation can be absorbed or decoupled by appropriate transformation. In the case of surface combustion it is sufficient to consider only the oxidizer diffusion equation among the species conservation equations. Of the remaining equations, Liu[1] has neglected the heat conduction in the solid. This is a serious omission, however, because it has been demonstrated previously,[2] for a similar problem, that solid phase heat transfer can be several orders of magnitude greater than gas phase heat transfer (principally due to the much greater density of the solid). This justifies

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* Postdoctoral Fellow of the National Science Foundation, Washington, D.C. Member AIAA.

† Forced convection is taken here to mean that part of the flow which is not associated with mass injection or moving boundaries.